Extracting azimuthal Fourier moments from sparse data

- Hermes data is always statistics-limited, because we always want to extract kinematic distributions in finer bins or more dimensions

⇨ Azimuthal Fourier moments must be extracted from sparse distributions containing bins with non-Gaussian statistics

- The acceptance can (and does!) cause a substantial systematic bias on observables extracted while integrating over kinematic variables on which that observable strongly depends. Examples:
  - P_t-weighted transverse target spin asymmetries
  - DVCS beam charge asymmetries (see next page)

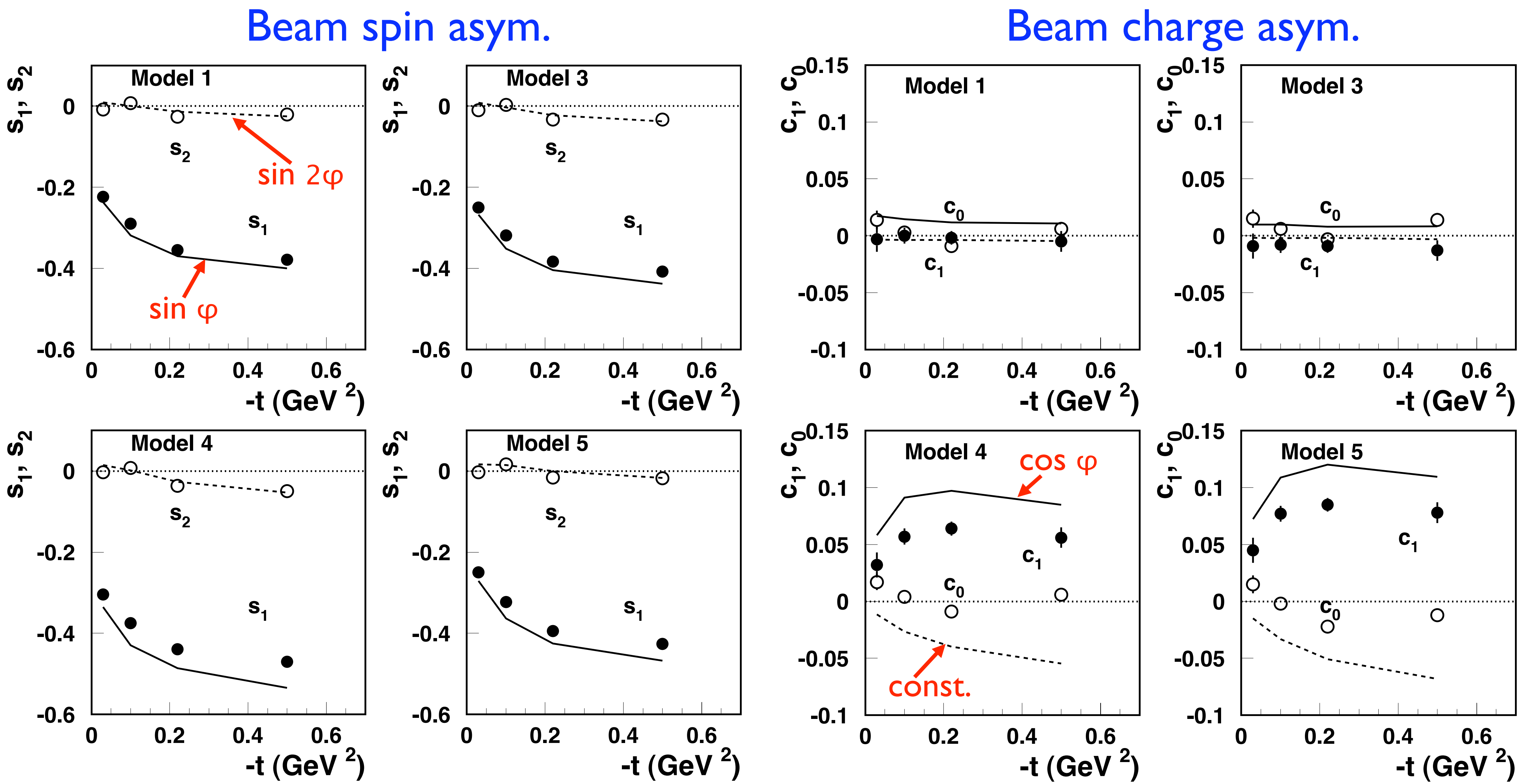
⇨ Quantify ignorance of full kinematic dependence, propagate to result

⇨ fit the full kinematic dependence on (x,y,z,P_t) using some standard set of 4D orthogonal functions, then fold with known $\sigma_{UU}(x,y,z,P_t)$

Covariance matrix of fitted coefficients is propagated through folding

Bernhard's Bad News -- the DVCS PEPSI Challenge

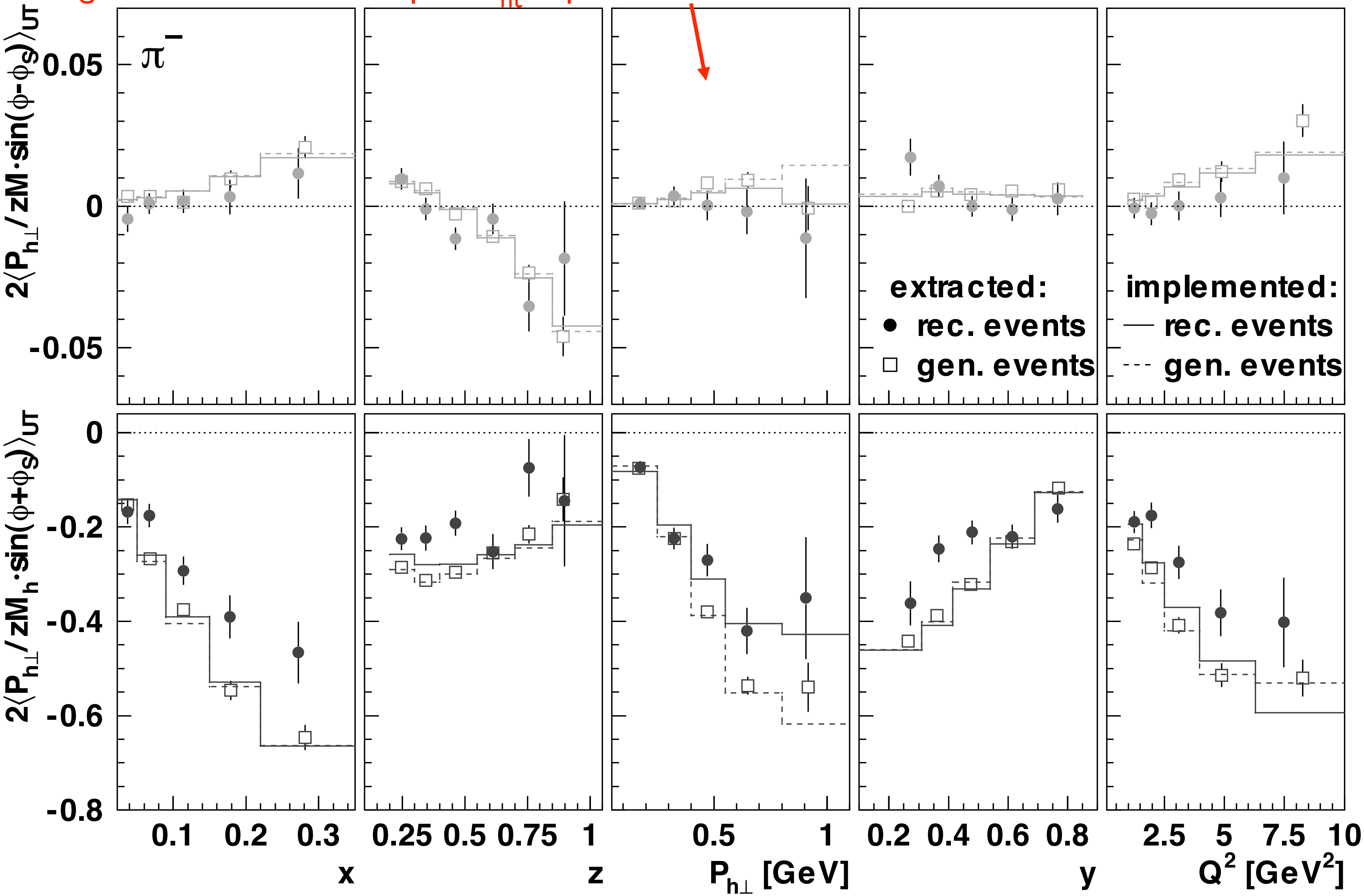
Curves: asymmetries of GPD models in MC, evaluated at bin center
Points: asym. extracted from MC data, averaged over most kinematics



Acceptance bias on integrated kinematic variables
on which the asymmetry strongly depends

Uli's Bad News -- the SIDIS gmc_trans Challenge

Ignore solid curves except in P_{ht} dependence



Acceptance bias on integrated kinematic variables
on which the asymmetry strongly depends

Maximum-Likelihood fit: unpol. example

For ML fit of unpol. azimuthal moments, the event dist'n and PDF are

$$C\,N(x,y,z,P_t,\phi,\phi_S) \;=\; \varepsilon(x,y,z,P_t,\phi,\phi_S)\,\underline{\sigma}_{UU}(x,y,z,P_t)\;\times$$
$$\left[1+A_{UU}^{\cos\phi}(\lambda_1,x,y,z,P_t)\,\cos\phi\;+\;A_{UU}^{\cos 2\phi}(\lambda_2,x,y,z,P_t)\,\cos(2\phi)\right]$$
$$\equiv\; F_{UU}(\lambda_1,\lambda_2,x,y,z,P_t,\phi,\phi_S)$$ (Probability Density Fun.)

Maximize Likelihood with respect to parameter sets λ_1, λ_2:

$$\mathcal{L}(\lambda_1,\lambda_2) \;=\; \frac{\prod_{i=1}^{N_x} F_{UU}(\lambda_1,\lambda_2,x_i,y_i,z_i,P_{ti},\phi_i,\phi_{Si})}{\mathcal{N}_{UU}^{N_x}(\lambda_1,\lambda_2)}$$

The denominator fixes the normalization of the PDF as the parameter sets λ_1 and λ_2 are stepped in the fit search:

$$\mathcal{N}_{UU}(\lambda_1,\lambda_2) \;=\; \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_t\,\mathrm{d}\phi\,\mathrm{d}\phi_S\; F_{UU}(\lambda_1,\lambda_2,x,y,z,P_t,\phi,\phi_S)$$

Acceptance ε and azimuthally averaged cross section $\underline{\sigma}_{UU}$ do not depend on the fitting parameter sets λ_1 and λ_2

⇨ **they can be omitted in calculation of the numerator!!**

How can we conveniently evaluate the normalization integral?

PDF Normalization: unpolarized case

Probability Density Function normalization:

$$\mathcal{N}_{UU}(\lambda_1,\lambda_2) = \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_t\,\mathrm{d}\phi\,\mathrm{d}\phi_S\;\varepsilon(x,y,z,P_t,\phi,\phi_S)\,\underline{\sigma}_{UU}(x,y,z,P_t)\,\times$$
$$\left[1+A_{UU}^{\cos\phi}(\lambda_1,x,y,z,P_t)\cos\phi+A_{UU}^{\cos 2\phi}(\lambda_2,x,y,z,P_t)\cos(2\phi)\right]$$

Solution:

Use Monte Carlo integration method with azimuthal event weights.

As Pythia MC events are distributed according to $\varepsilon\underline{\sigma}_{UU}$, PDF integral is

$$\mathcal{N}_{UU}(\lambda_1,\lambda_2) = \sum_{j=1}^{N_{MC}} W_j^{MC}\left[1 + A_{UU}^{\cos\phi}(\lambda_1,x_j,y_j,z_j,P_{tj})\cos\phi_j + A_{UU}^{\cos 2\phi}(\lambda_2,x_j,y_j,z_j,P_{tj})\cos(2\phi_j)\right]$$

For efficiency:

All factors in both likelihood product (expt'l events) and integral sum (MC events) can be tabulated for all events before starting the fit search

Result of the fit

$$\sigma_{UU}(x,y,z,P_t,\phi) = \underline{\sigma}_{UU}(x,y,z,P_t) \times \left[1 + A_{UU}^{\cos\phi}(\lambda_1,x,y,z,P_t)\cos\phi + A_{UU}^{\cos 2\phi}(\lambda_2,x,y,z,P_t)\cos(2\phi)\right]$$

The parameter sets λ_1 and λ_2 could be archived in the Durham data base,

but we compare models to asymmetries in yields integrated over some variables:

$$\langle\cos\phi\rangle_{UU}^{h}(x) = \frac{\int \mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_t\ \underline{\sigma}_{UU}^{Born}(x,y,z,P_t)\ A_{UU}^{\cos\phi}(\lambda_1,x,y,z,P_t)}{\int \mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_t\ \underline{\sigma}_{UU}^{Born}(x,y,z,P_t)}$$

This integral can be evaluated using $\underline{\sigma}_{UU}(x,y,z,P_t)$ from parton dist. funs and

measured hadron multiplities, or a Pythia MC event set generated in 4π:

$$\langle\cos\phi\rangle_{UU}^{h}(x) = \sum_{j=1}^{N_{MC}} W_j^{MC} A_{UU}^{\cos\phi}(\lambda_1,x_j,y_j,z_j,P_{tj}) / \sum_{j=1}^{N_{MC}} W_j^{MC}$$

If the parameterization is linear in the fitted parameters,
it is easy to propagate their covariance matrices through this sum

Exchange unknown systematic error for well-defined statistical uncertainty

Maximum-likelihood fit: transverse-polarized case

Using the predetermined full kinematic dependence of the $\cos(n\phi)$ moments, the event distribution and PDF for target polarization dist'n $\rho(P)$, $-1<P<1$, is:

$$\begin{aligned} C\, N(P,x,y,z,P_t,\phi,\phi_S) \;=\;& \rho(P)\; \varepsilon(x,y,z,P_t,\phi,\phi_S)\; \underline{\sigma}_{UU}(x,y,z,P_t) \times \\ & \Big\{ 1 + A_{UU}^{\cos\phi}(x,y,z,P_t)\cos\phi + A_{UU}^{\cos 2\phi}(x,y,z,P_t)\cos(2\phi) \\ & \quad + P\left[A_C(\lambda_1,x,y,z,P_t)\sin(\phi+\phi_S) + A_S(\lambda_2,x,y,z,P_t)\sin(\phi-\phi_S)\right] \Big\} \\ \equiv\;& F(\lambda_1,\lambda_2,P,x,y,z,P_t,\phi,\phi_S) \end{aligned}$$

ML treats the target polarization P like any other (e.g., kinematic) variable.

Again the parameter-independent factor $\varepsilon\underline{\sigma}_{UU}$ can be omitted

in the numerator of the Likelihood:

$$\mathcal{L}(\lambda_1,\lambda_2) \;=\; \prod_{i=1}^{N} \frac{F(\lambda_1,\lambda_2,P_i,x_i,y_i,z_i,P_{ti},\phi_i,\phi_{Si})^{W_i}}{\mathcal{N}(\lambda_1,\lambda_2)^{W_i}}$$

Here, W_i are event weights (from e.g. RICH PId)

The product in the denominator is independent of λ_1 and λ_2 and can be ignored in the likelihood maximization, if the whole data set has no net polarization:

$$\int \mathrm{d}P\; P\,\rho(P) = 0$$

PDF normalization: transverse-polarized case

In the PDF normalization integral, the integration over P factorizes:

$$\mathcal{N}(\lambda_1,\lambda_2) = \int dP\, dx\, dy\, dz\, dP_t\, d\phi\, d\phi_S\; \rho(P)\; \varepsilon(x,y,z,P_t,\phi,\phi_S)\; \underline{\sigma}_{UU}(x,y,z,P_t) \times$$
$$\Big\{1 + A_{UU}^{\cos\phi}(x,y,z,P_t)\cos\phi + A_{UU}^{\cos 2\phi}(x,y,z,P_t)\cos(2\phi)$$
$$+ P\left[A_C(\lambda_1,x,y,z,P_t)\sin(\phi+\phi_S) + A_S(\lambda_2,x,y,z,P_t)\sin(\phi-\phi_S)\right]\Big\}$$
$$= \int dP \rho(P) \cdot \int dx\, dy\, dz\, dP_t\, d\phi\, d\phi_S\; \varepsilon(x,y,z,P_t,\phi,\phi_S)\; \underline{\sigma}_{UU}(x,y,z,P_t) \times$$
$$\Big\{1 + A_{UU}^{\cos\phi}(x,y,z,P_t)\cos\phi + A_{UU}^{\cos 2\phi}(x,y,z,P_t)\cos(2\phi)$$
$$+ \frac{\int dP\, P\rho(P)}{\int dP\, \rho(P)}\left[A_C(\lambda_1,x,y,z,P_t)\sin(\phi+\phi_S) + A_S(\lambda_2,x,y,z,P_t)\sin(\phi-\phi_S)\right]\Big\}$$

This normalization integral is obviously independent of λ_1 and λ_2 if...

$$\int dP\, P\rho(P) = 0$$

If necessary, this might be arranged by scaling the weights of events recorded with one polarization sign.

ML fit: DVCS beam-helicity asymmetry

The event distribution and PDF for beam polarization dist'n $\rho(P)$, $-1<P<1$, is:

$$\begin{aligned} C\,N(P,x,y,t,\phi) &= \rho(P)\,\varepsilon(x,y,t,\phi)\,\underline{\sigma}_{UU}(x,y,t)\times \\ &\quad \{1+P[A_1(\lambda_1,x,y,t)\sin(\phi)+A_2(\lambda_2,x,y,t)\sin(2\phi)]\} \\ &\equiv F(\lambda_1,\lambda_2,P,x,y,t,\phi) \end{aligned}$$

Again the parameter-independent factor $\varepsilon\underline{\sigma}_{UU}$ can be omitted in the numerator of the Likelihood:

$$\mathcal{L}(\lambda_1,\lambda_2) = \prod_{i=1}^{N} \frac{F(\lambda_1,\lambda_2,P_i,x_i,y_i,t_i,\phi_i)^{W_i}}{\mathcal{N}(\lambda_1,\lambda_2)^{W_i}}$$

Here, W_i are event weights (from e.g. RICH PId)

We will show the normalization in the denominator is independent of λ_1 and λ_2 and can therefore be ignored in the likelihood maximization if either:

-- the net target polarization for the whole data set is zero:

$$\int \mathrm{d}P\, P\,\rho(P) = 0$$

-- or if the acceptance has no odd harmonics in ϕ:

$$\varepsilon(x,y,t,\phi) = \varepsilon(x,y,t,-\phi)$$

PDF normalization: DVCS beam-helicity case

In the PDF normalization integral, the integration over P factorizes:

$$\mathcal{N}(\lambda_1,\lambda_2) = \int \mathrm{d}P\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t\,\mathrm{d}\phi\;\rho(P)\;\varepsilon(x,y,t,\phi)\;\underline{\sigma}_{UU}(x,y,t)\times$$

$$\{1 + P\,[\,A_1(\lambda_1,x,y,t)\sin(\phi) + A_2(\lambda_2,x,y,t)\sin(2\phi)]\}$$

$$= \int \mathrm{d}P\,\rho(P)\;\cdot\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t\,\mathrm{d}\phi\;\varepsilon(x,y,t,\phi)\;\underline{\sigma}_{UU}(x,y,t)\times$$

$$\left\{1 + \frac{\int \mathrm{d}P\,P\,\rho(P)}{\int \mathrm{d}P\,\rho(P)}\,[\,A_1(\lambda_1,x,y,t)\sin(\phi) + A_2(\lambda_2,x,y,t)\sin(2\phi)]\right\}$$

This normalization integral is independent of λ_1 and λ_2 if either...

$$\int \mathrm{d}P\,P\,\rho(P) = 0$$

(If necessary, this can be arranged by scaling the weights of events recorded with one polarization sign)

or if...

$$\varepsilon(x,y,t,\phi) = \varepsilon(x,y,t,-\phi)$$

because its convolution with sin(nϕ) again yields zero for the second term.

gmc_trans Challenge

Recent History

- (Un)binned Maximum-Likelihood fits to azimuthal Fourier amplitudes are significantly superior to least-χ^2 fits for data sets with few events
- Such Maximum-Likelihood fits are easy to implement
- ML fits should be a normal method for the binned presentation used up to now (A χ^2 fit should always be done as a sanity check)

News

- 4D fits to Collins, Sivers dependence on (x,y,z,P_t) given by gmc_trans models
- Uses 4D Taylor expansion to second order of each variable e.g. x^2zP_t, $z^2P_t^2$
- Fold these fits in Hermes acceptance with unpolarized cross section in 4π
- Uses distribution of gmc_trans events to represent unpolarized cross section
- Do the results agree with gmc_trans model asymmetries?

Folding the 4D fits

Fits: the full kinematic dependence of $A_C(x,y,z,P_t)$ and $A_S(x,y,z,P_t)$

- Only the leading terms in the 4D expansions will be well determined
- Will get weaker constraints on detailed dependence with correlations
- Convolution of fitted asymmetries with unpolarized Born cross section can yield asymmetry in any moment, e.g. the P_t -weighted asymmetry:

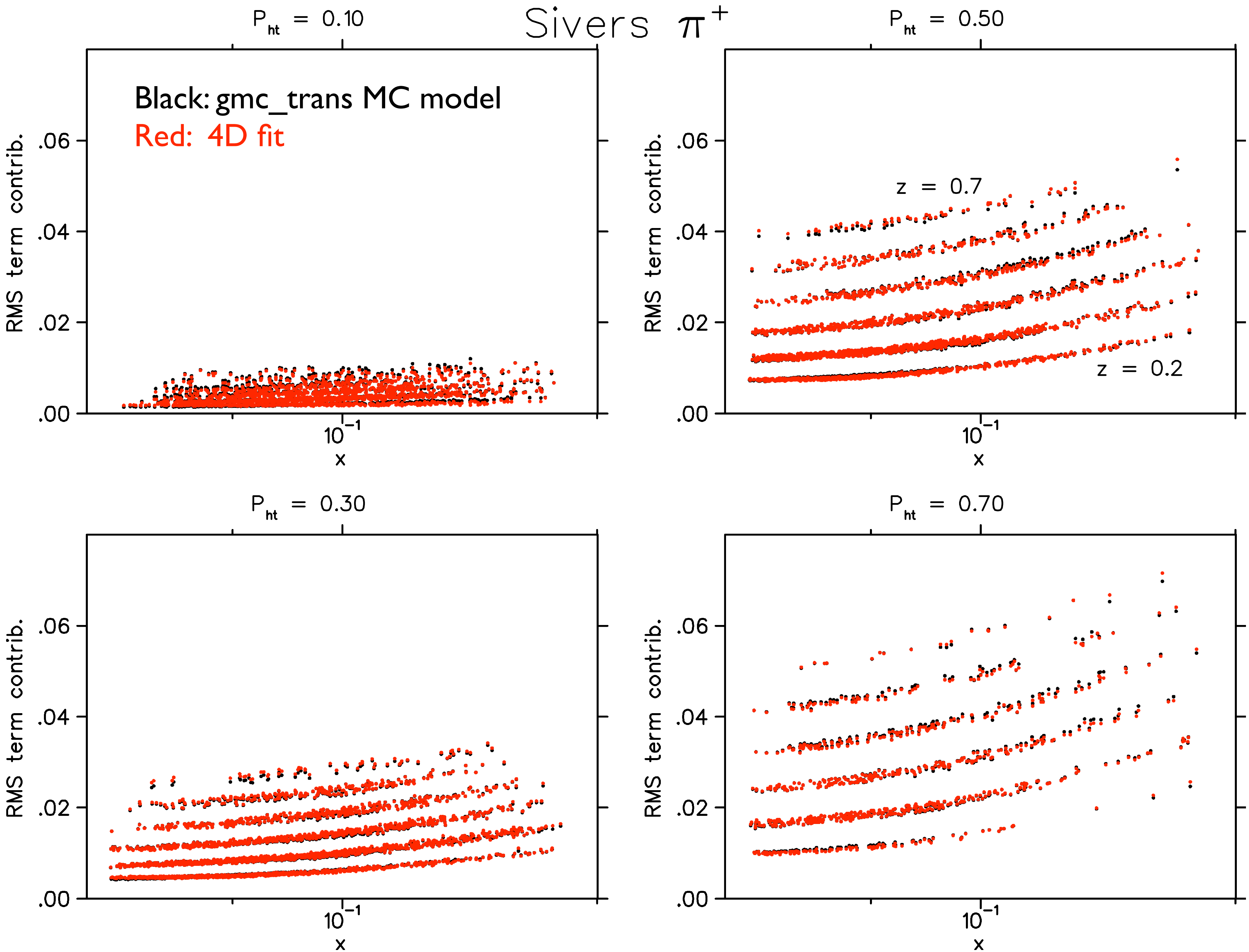
$$\langle \frac{P_t}{zM_\pi} \sin(\phi+\phi_S) \rangle_{UT}(x) = \frac{\int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}P_t\; P_t/(zM_\pi)\; \underline{\sigma}_{UU}^{Born}(x,y,z,P_t)\; A_C(\lambda_1, x, y, z, P_t)}{\int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}P_t\; \underline{\sigma}_{UU}^{Born}(x,y,z,P_t)}$$

Using unpolarized MC events generated in 4π :

$$\langle \frac{P_t}{zM_\pi} \sin(\phi+\phi_S) \rangle_{UT}(x) = \frac{\sum_{j=1}^{N_{MC}} W_j^{MC}\; P_{ti}/(z_i M_\pi)\; A_C(\lambda_1, x_j, y_j, z_j, P_{tj})}{\sum_{j=1}^{N_{MC}} W_j^{MC}}$$

Must propagate λ_1 covariance matrix through integral

44-parameter 4D fit to fully differential mcUser moment



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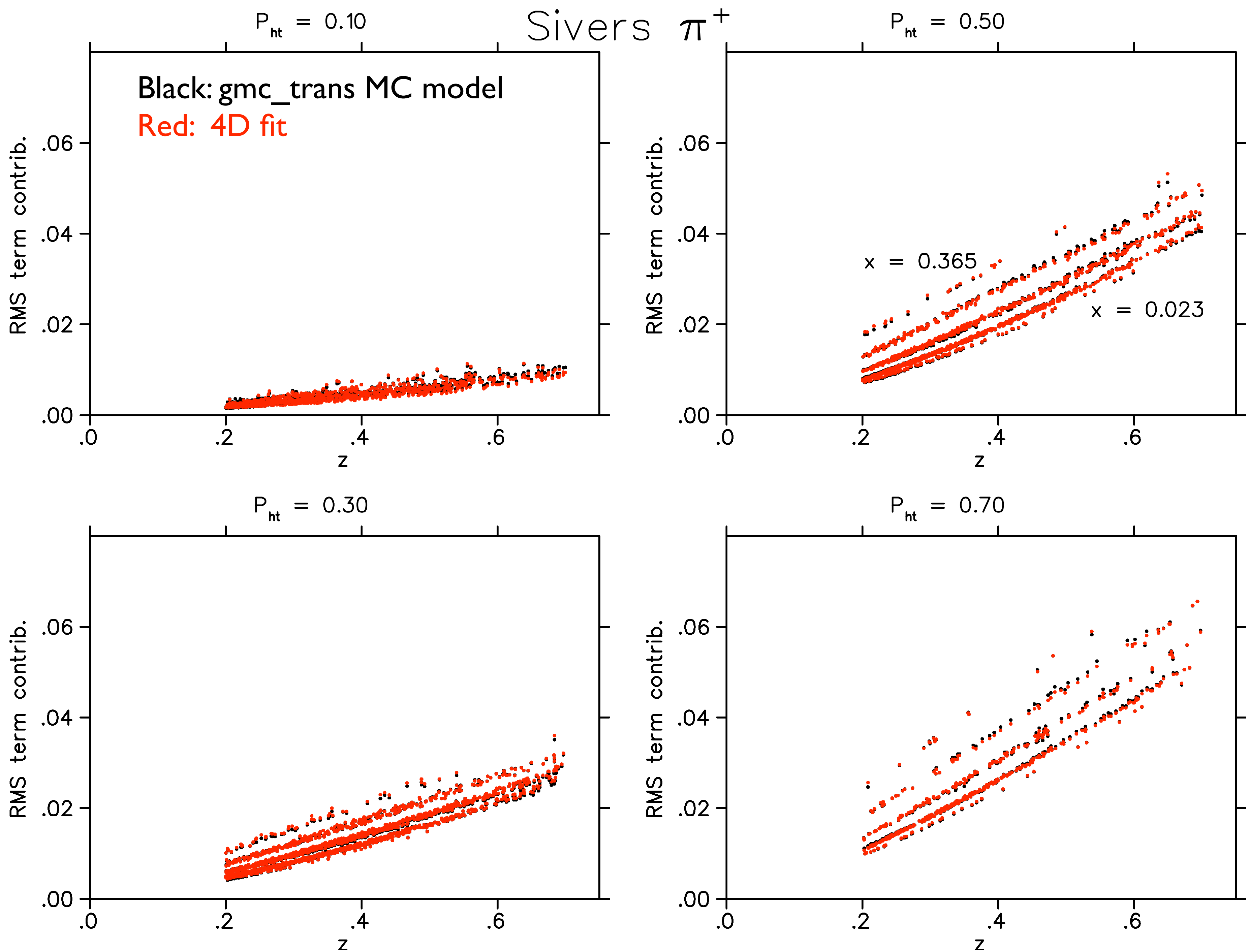

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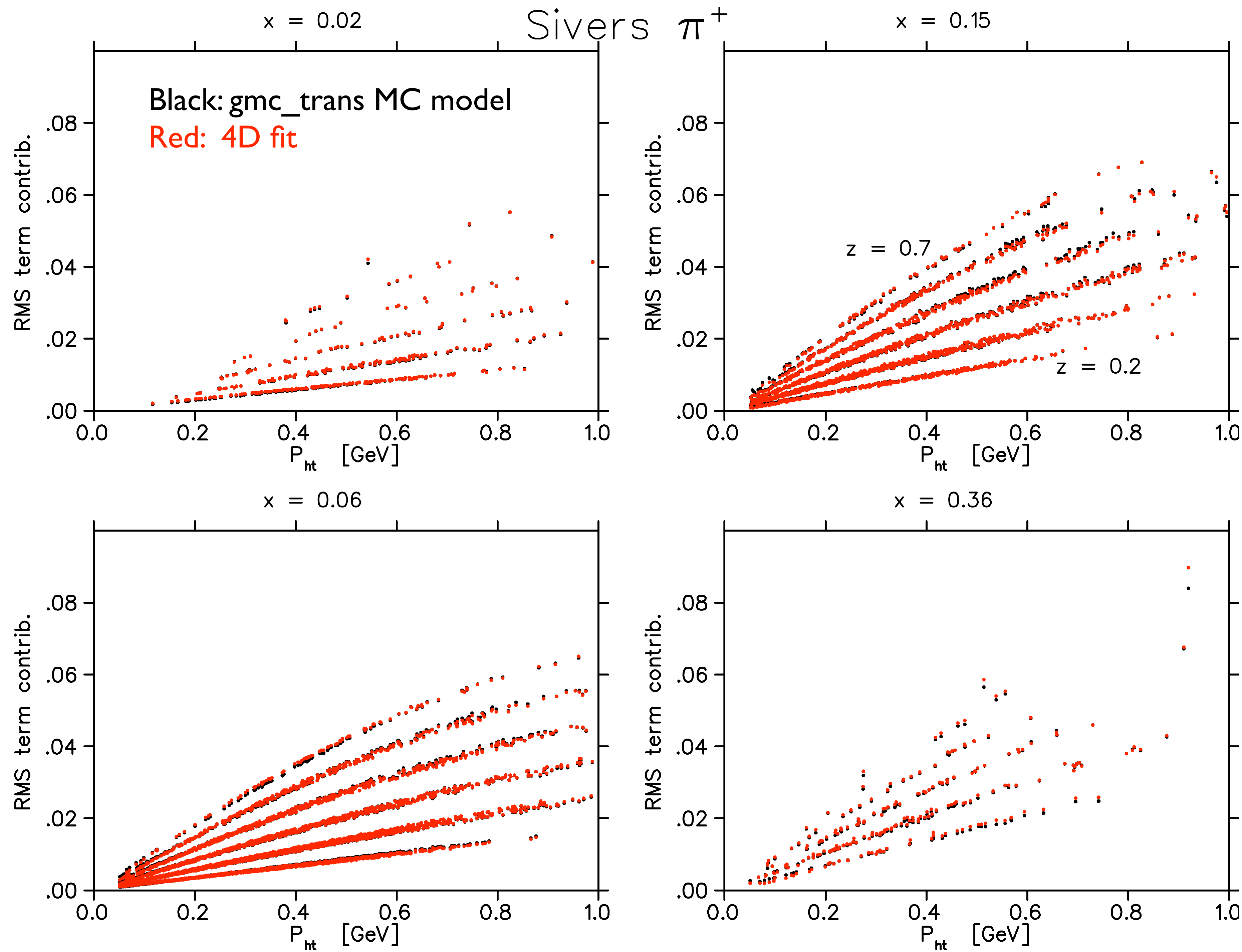

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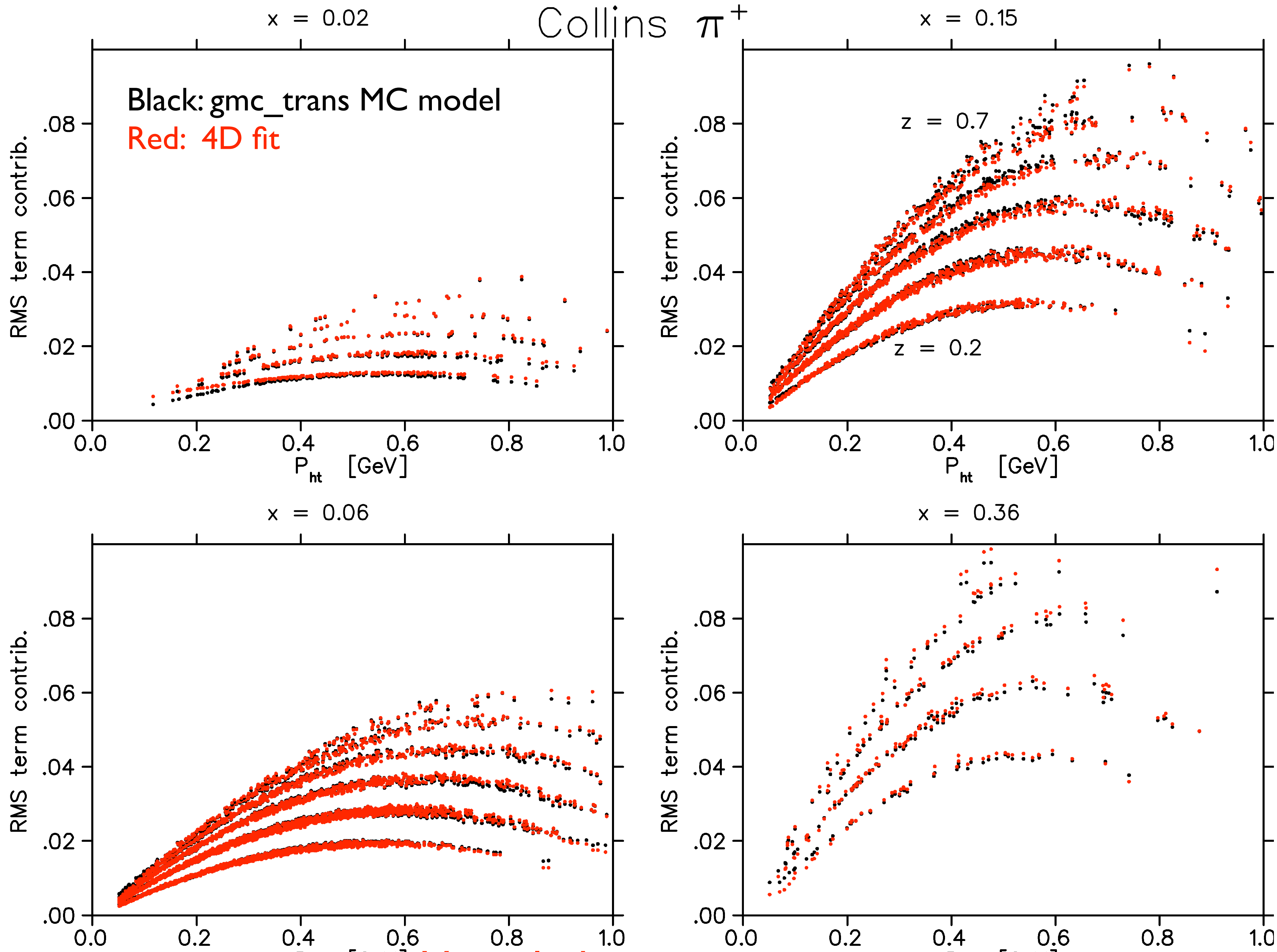

Virtual-photon asymmetries

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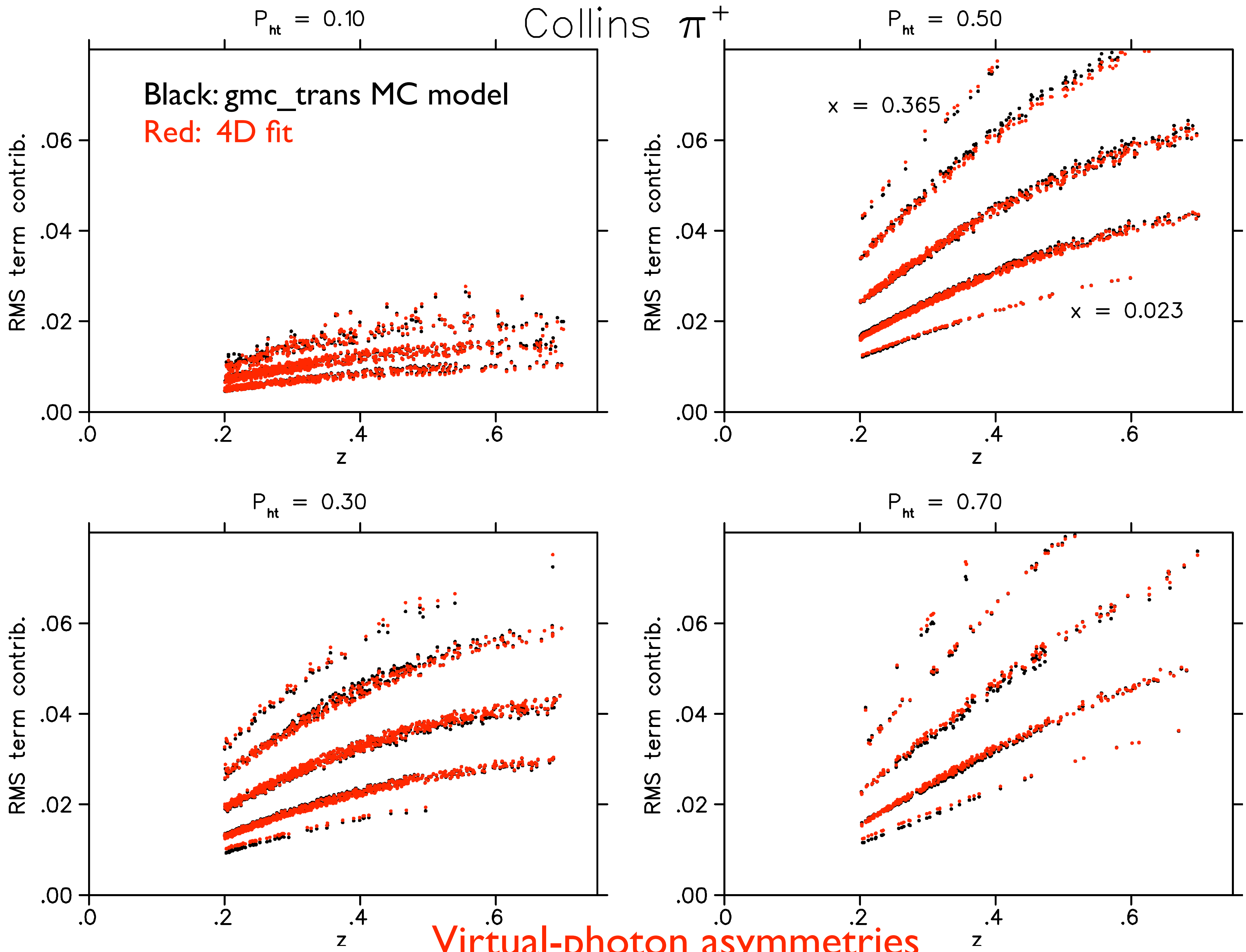

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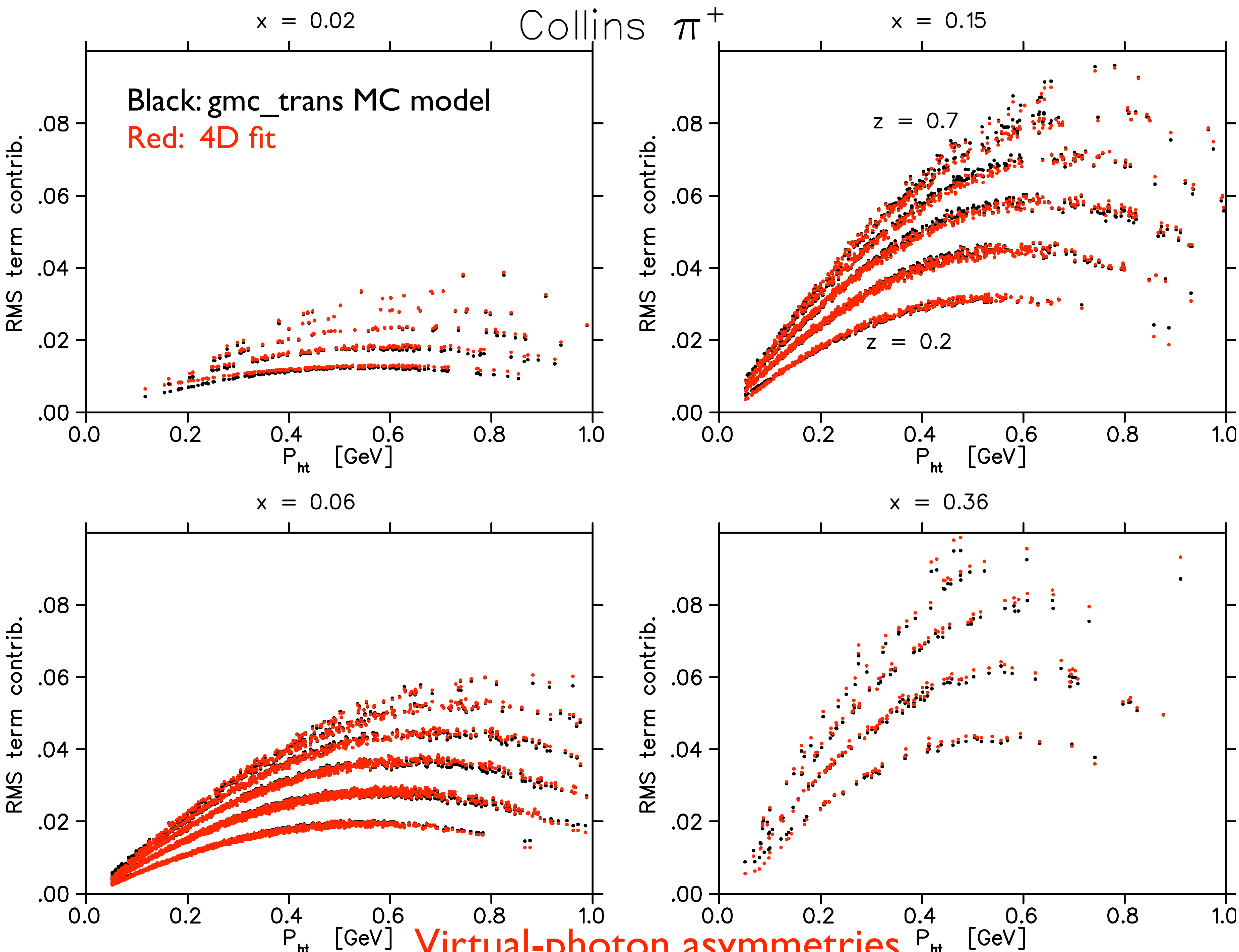

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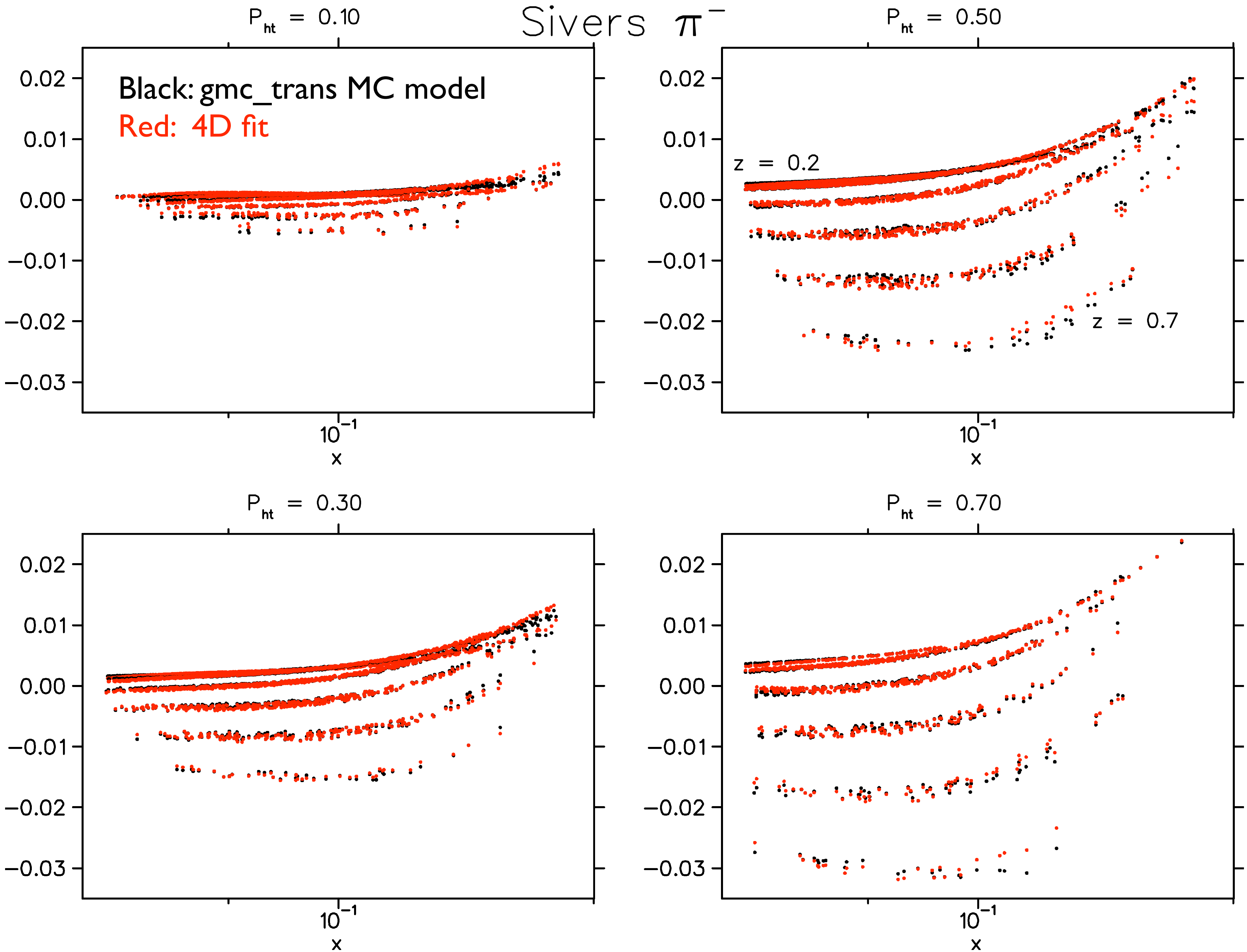

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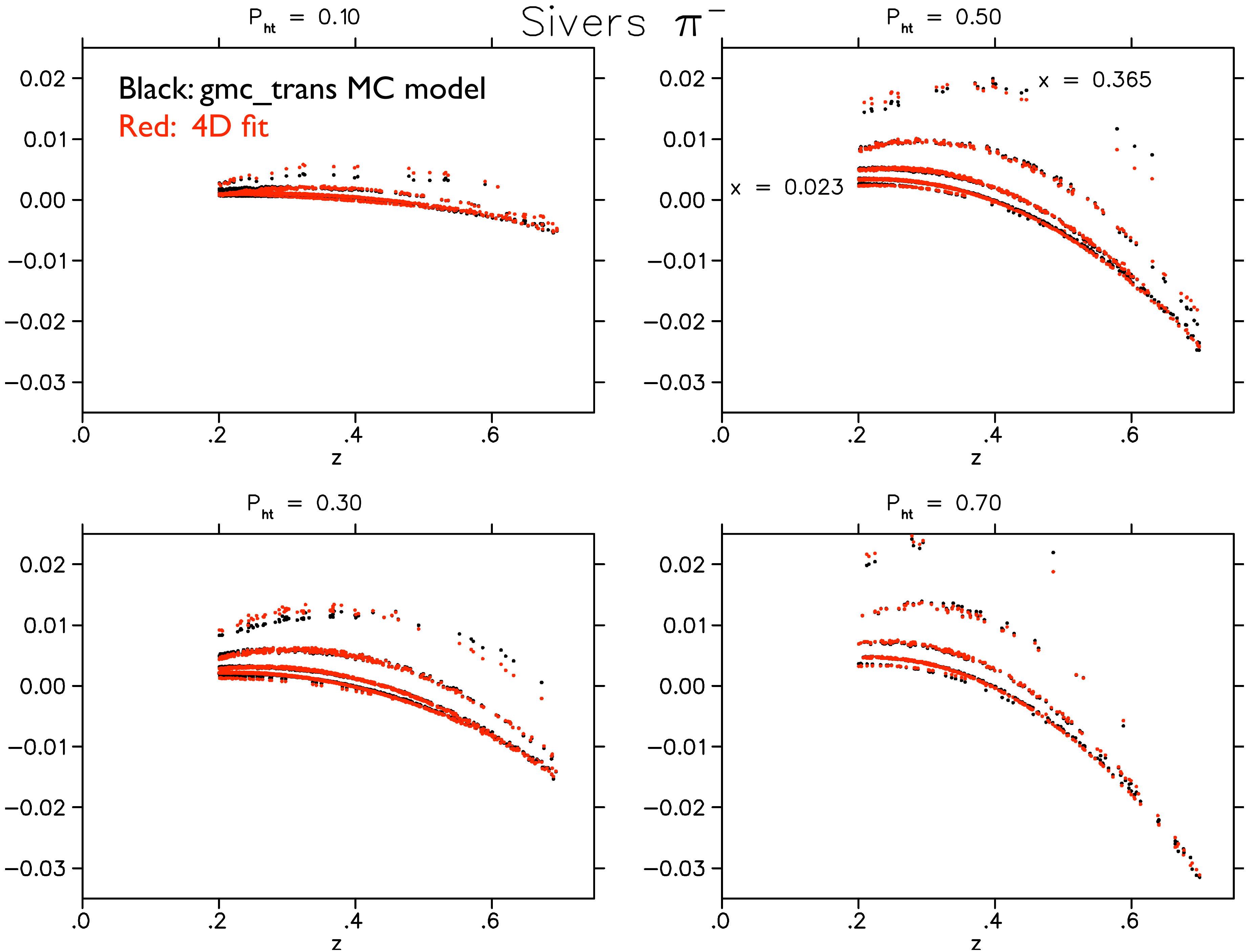

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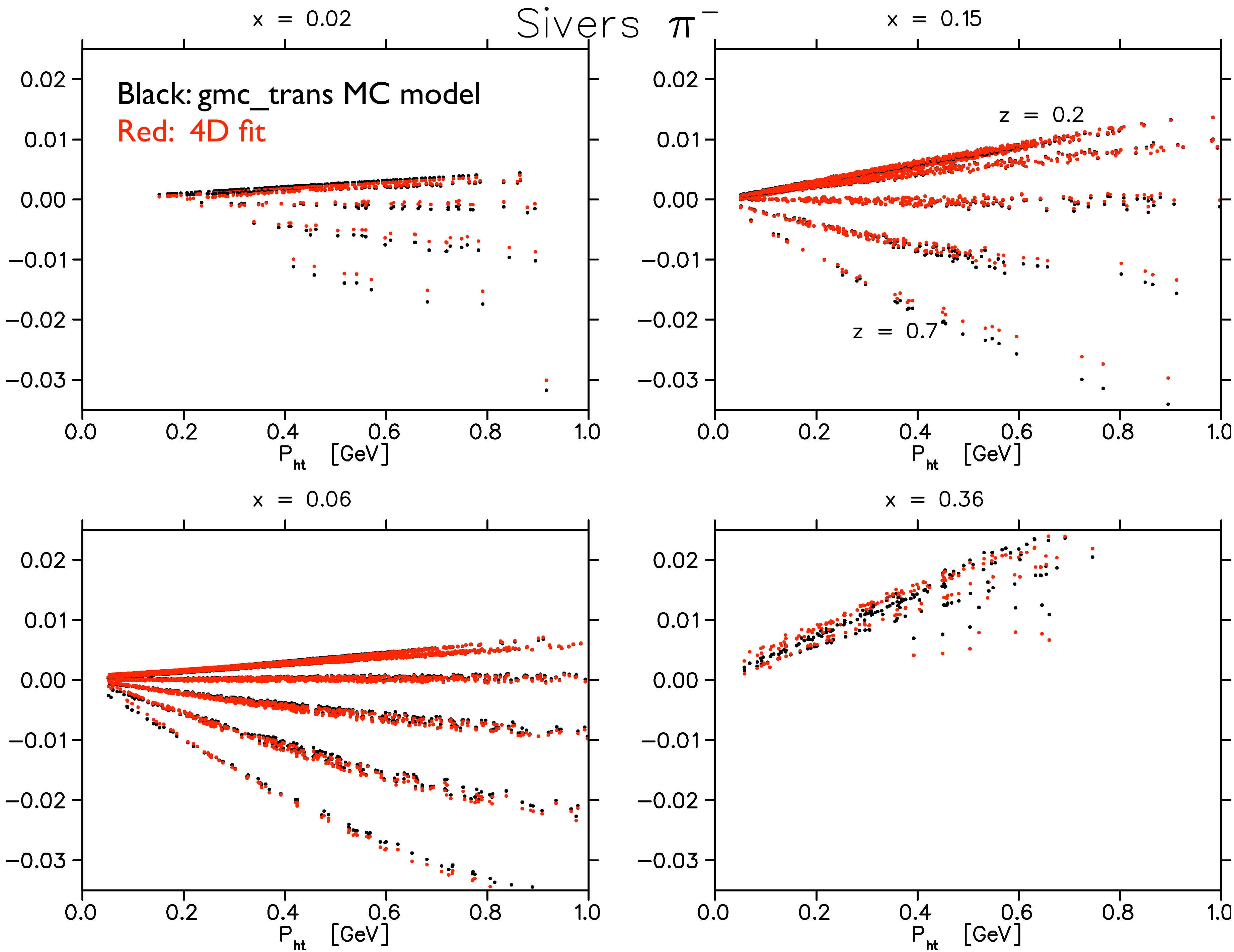

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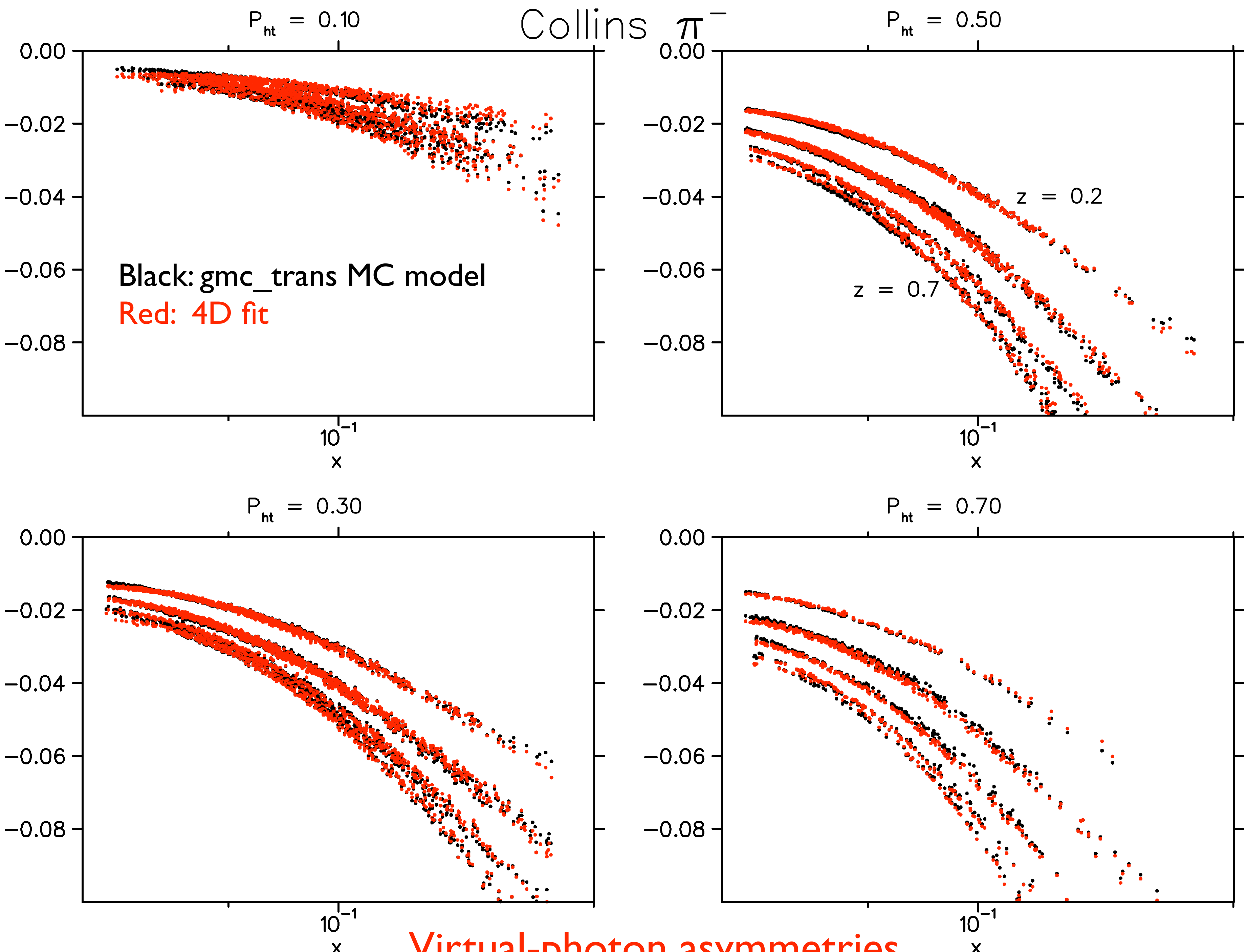

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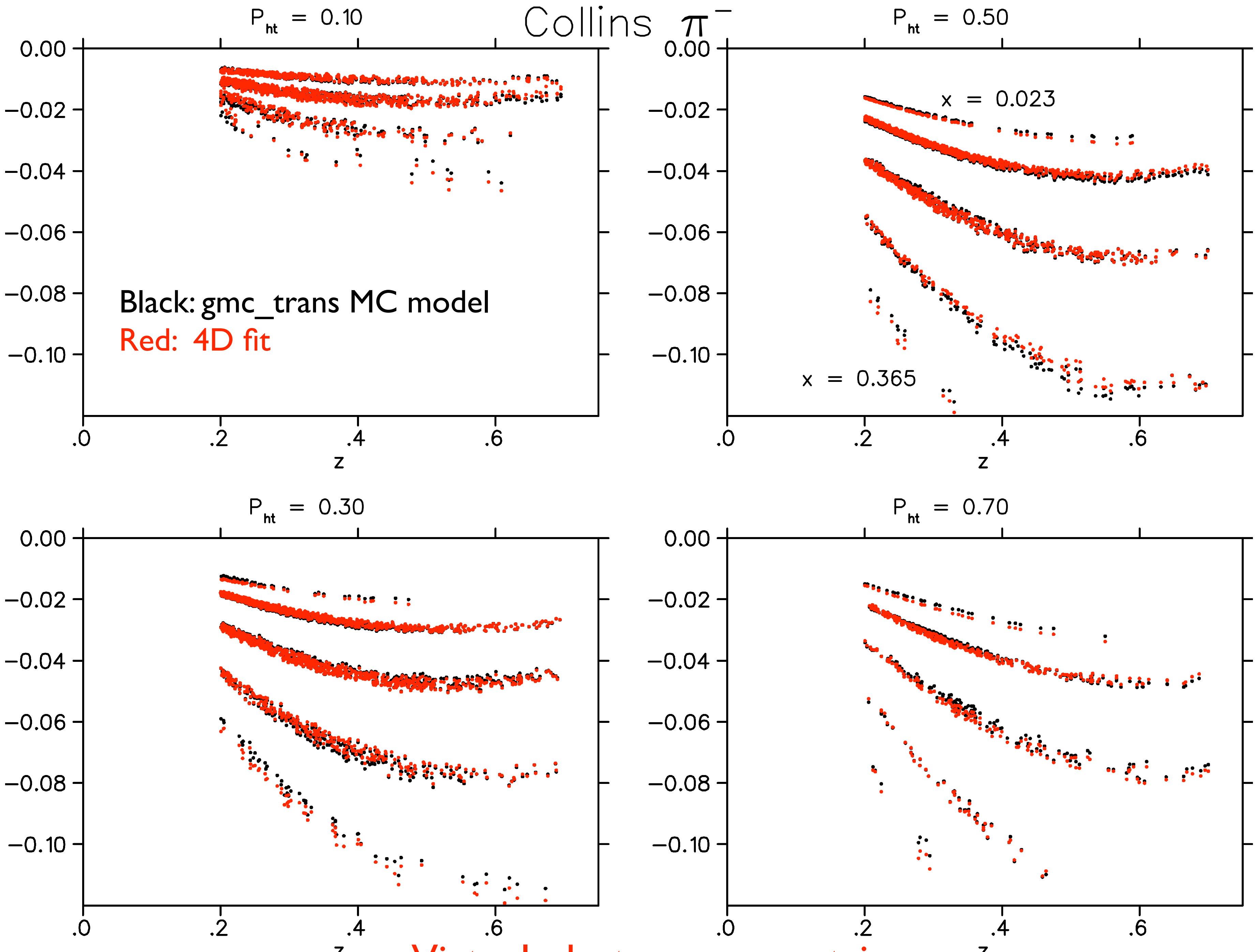

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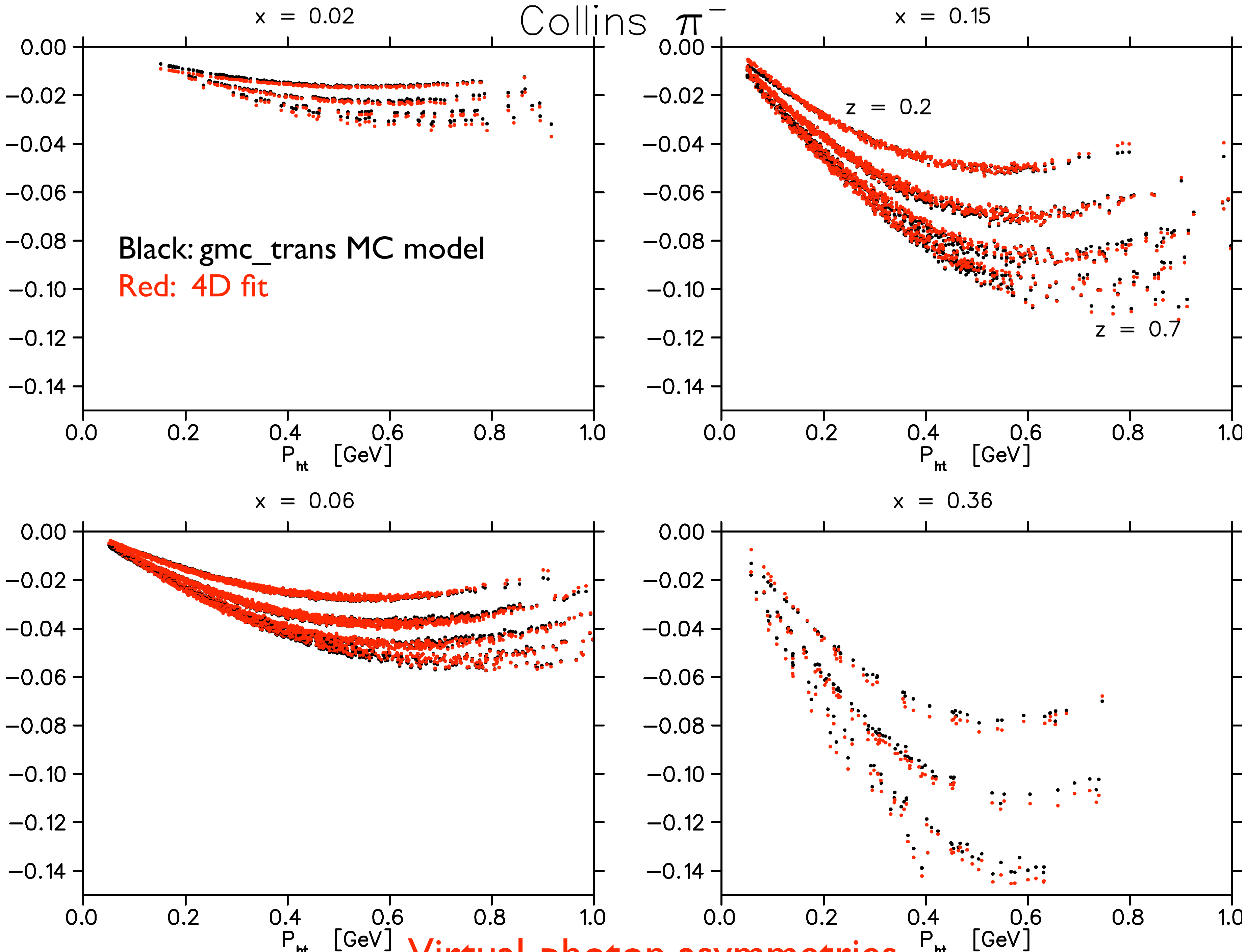
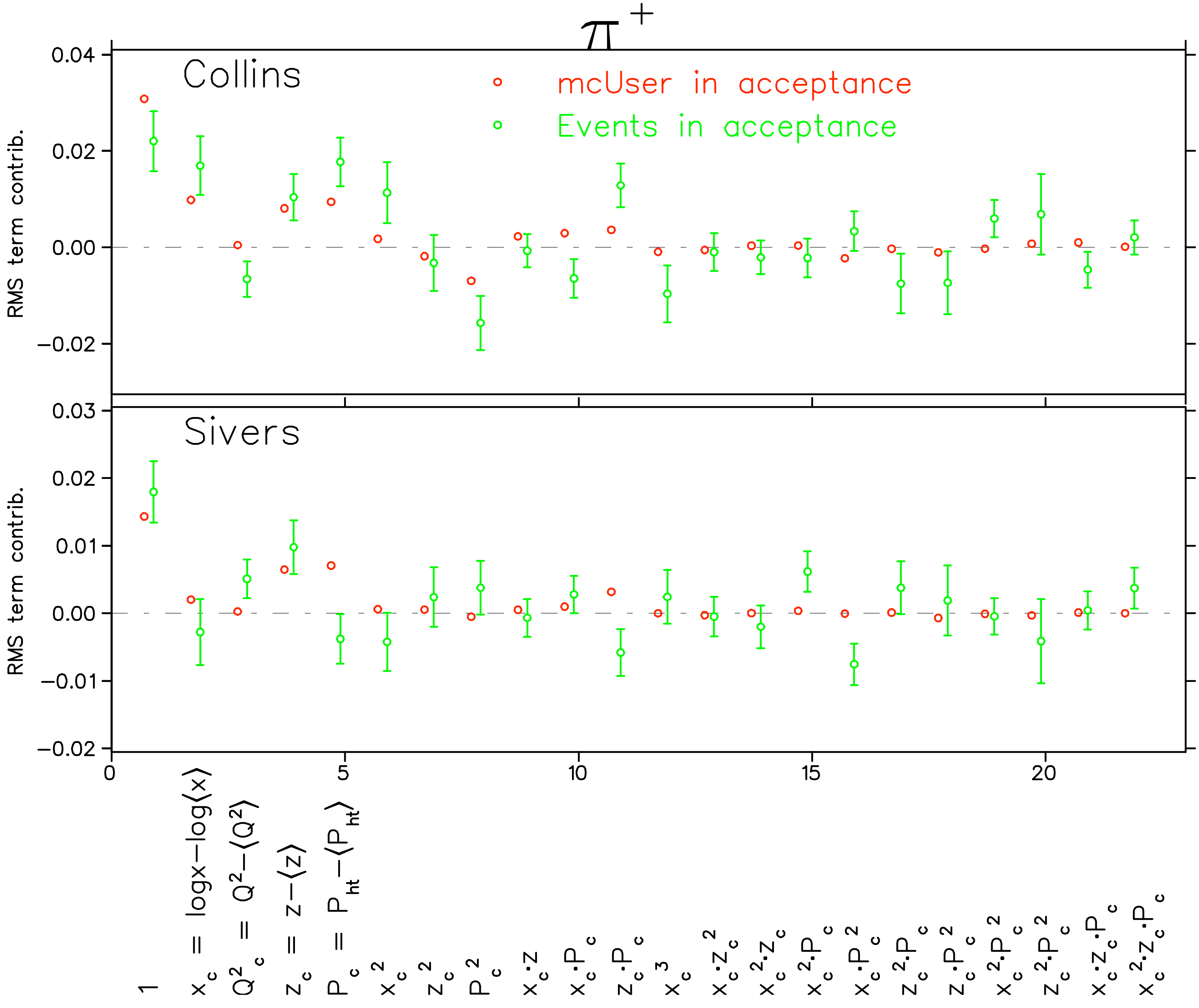


Virtual-photon asymmetries

Compare 4D fit of gmc_trans events to 4D fit of mcUser values



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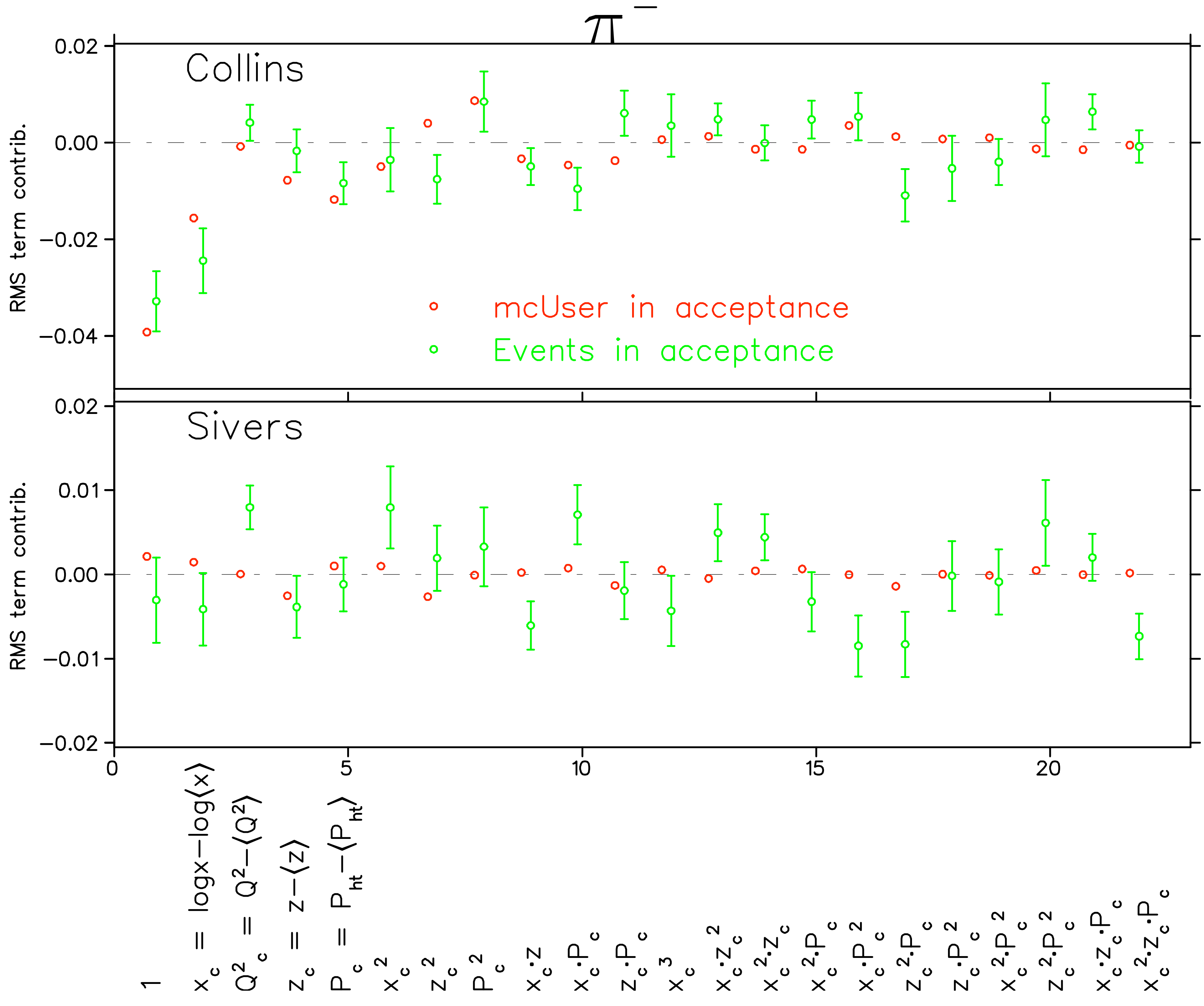
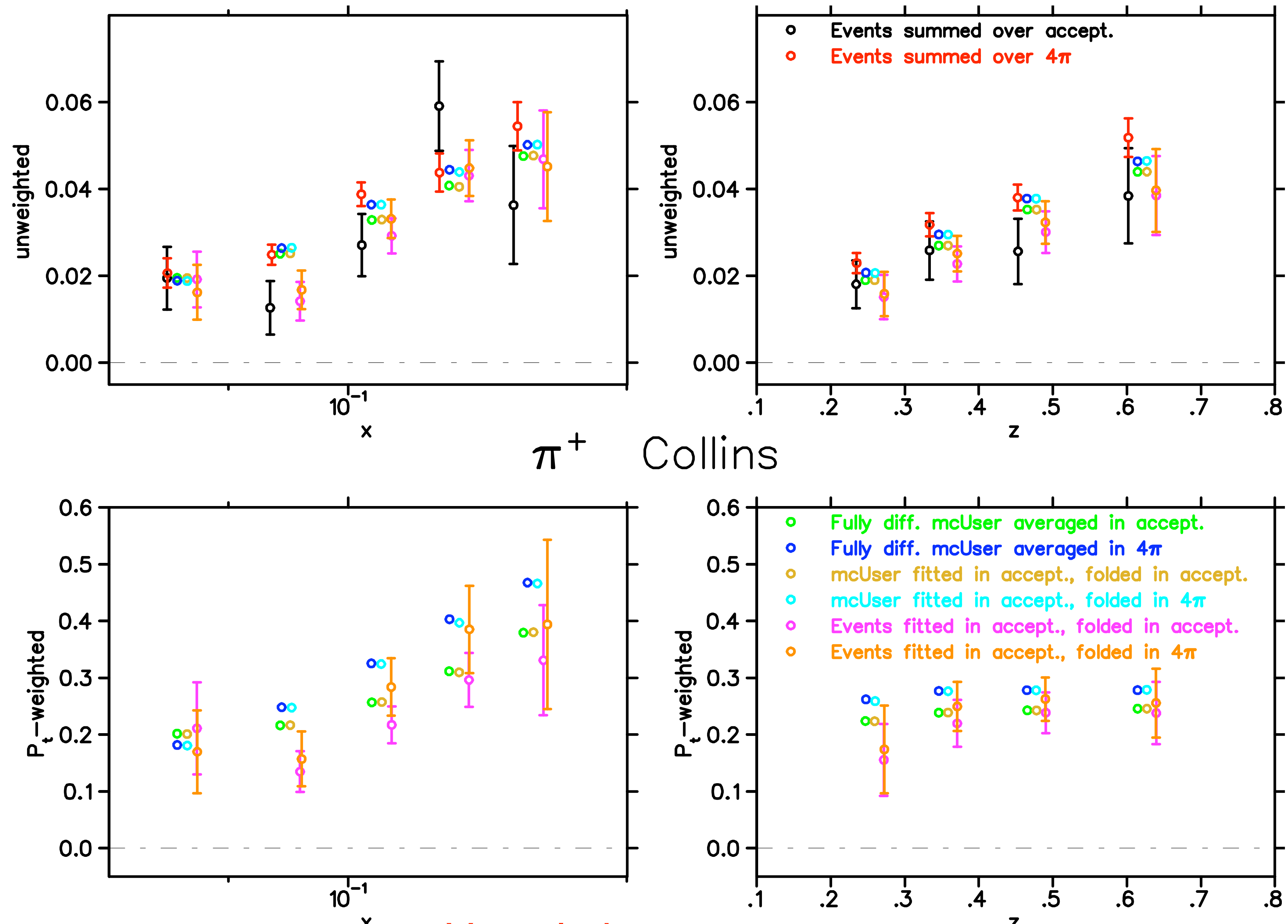


Compare standard extraction with 4D fit of gmc_trans mcUser values or events, folded with yield in acceptance or 4π



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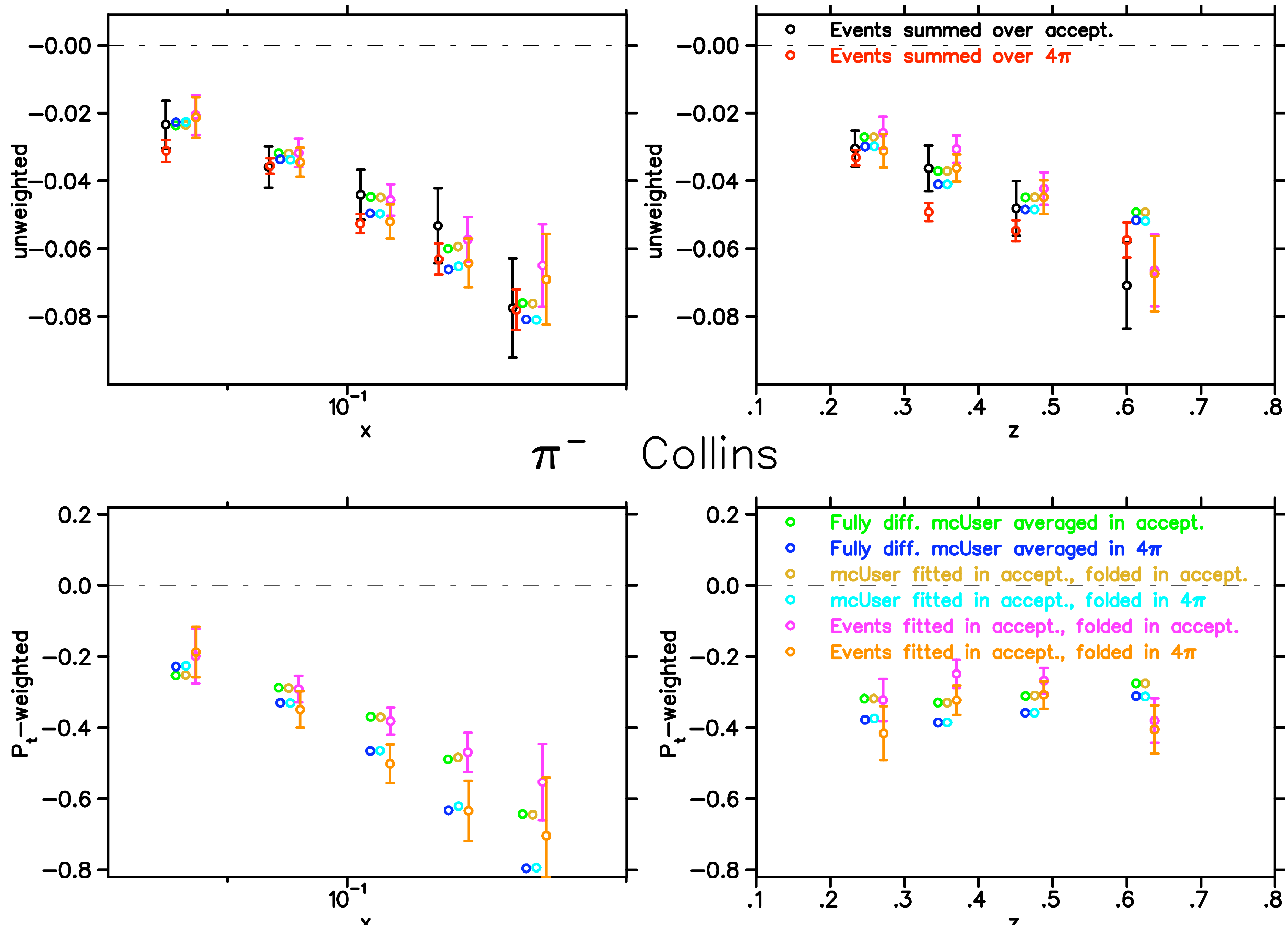

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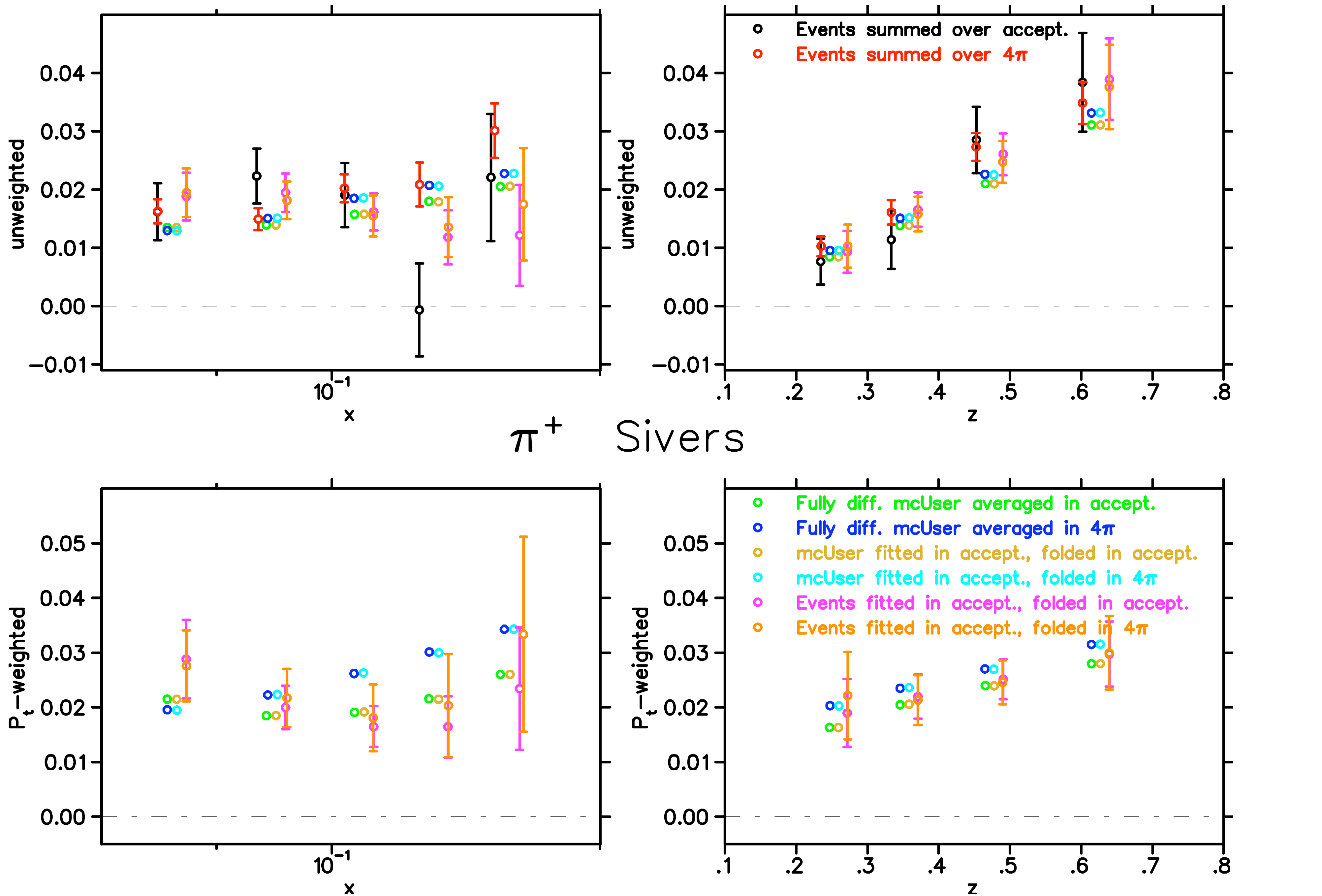
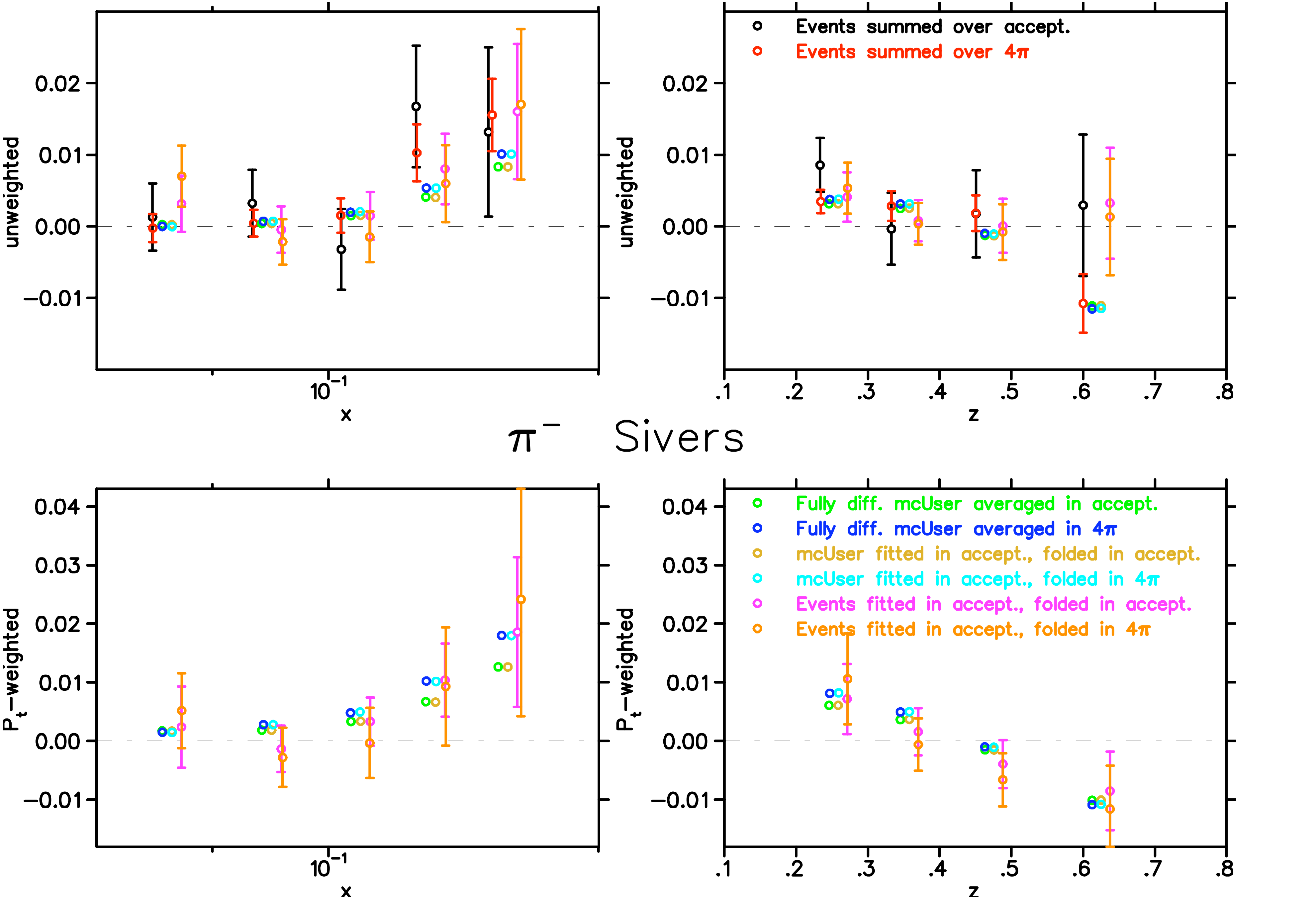
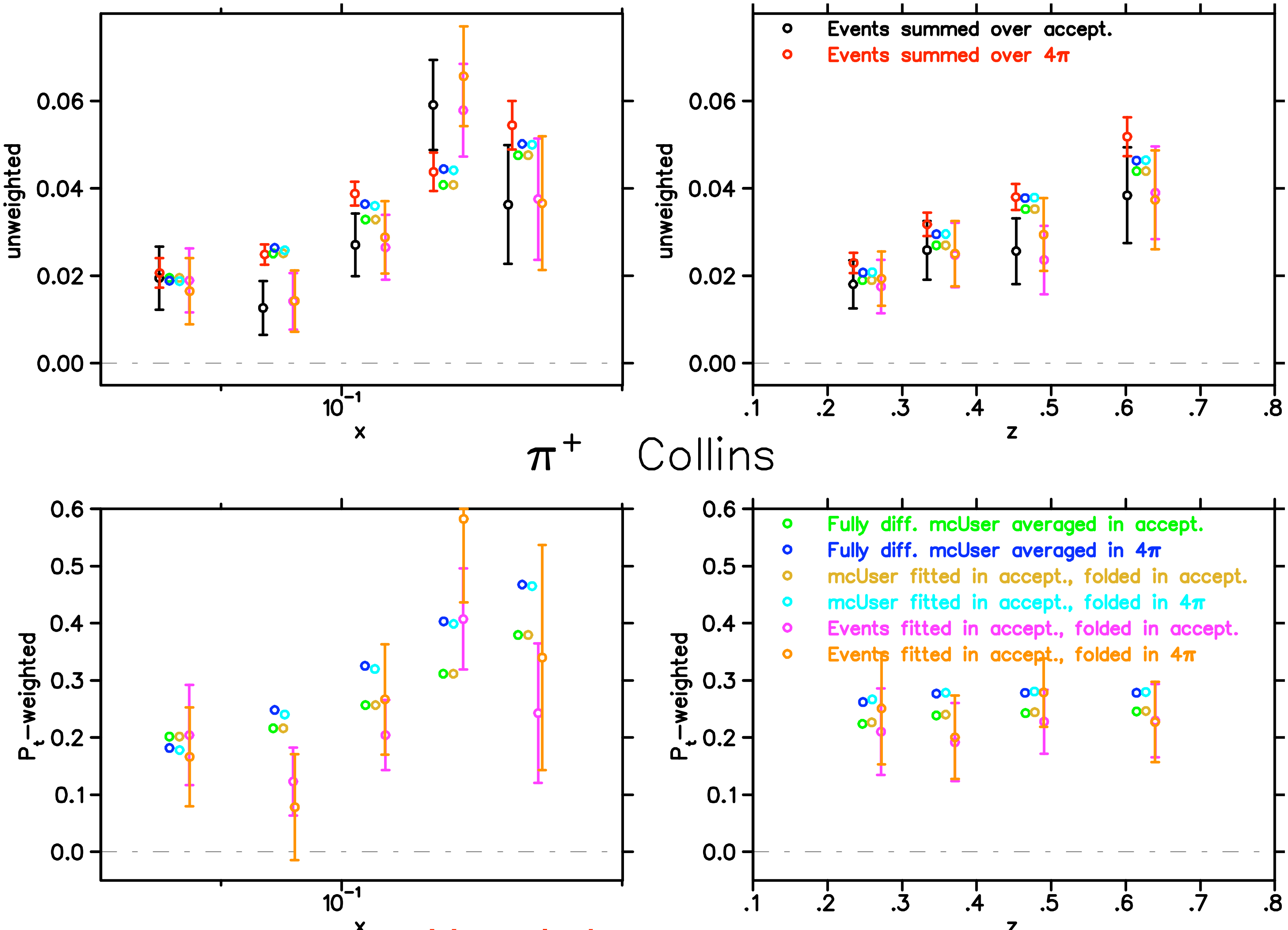


Compare standard extraction with 4D fit of gmc_trans mcUser values or events, folded with yield in acceptance or 4π



Compare standard extraction with 3D fit of gmc_trans mcUser values or events, folded with yield in acceptance or 4π



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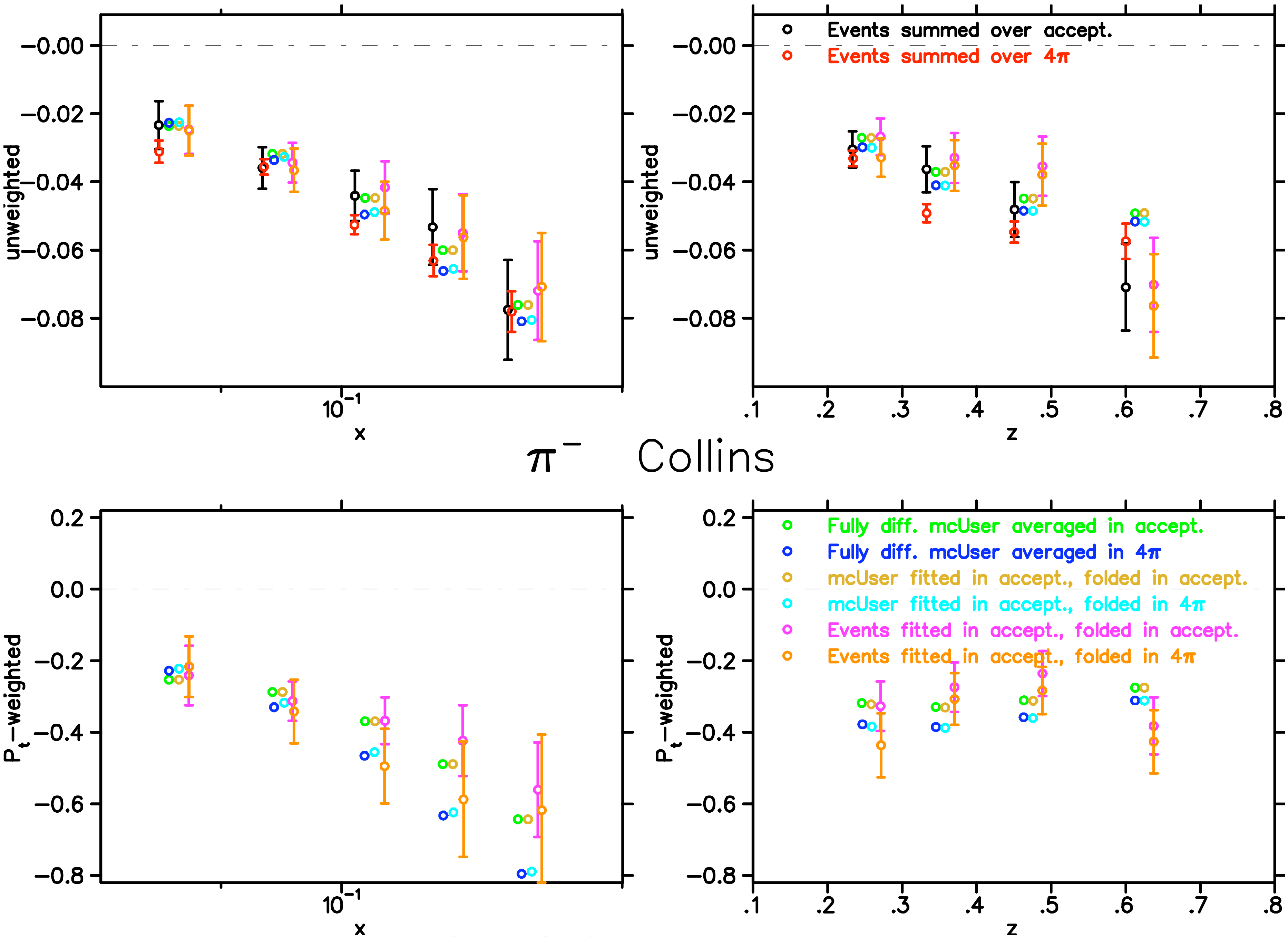

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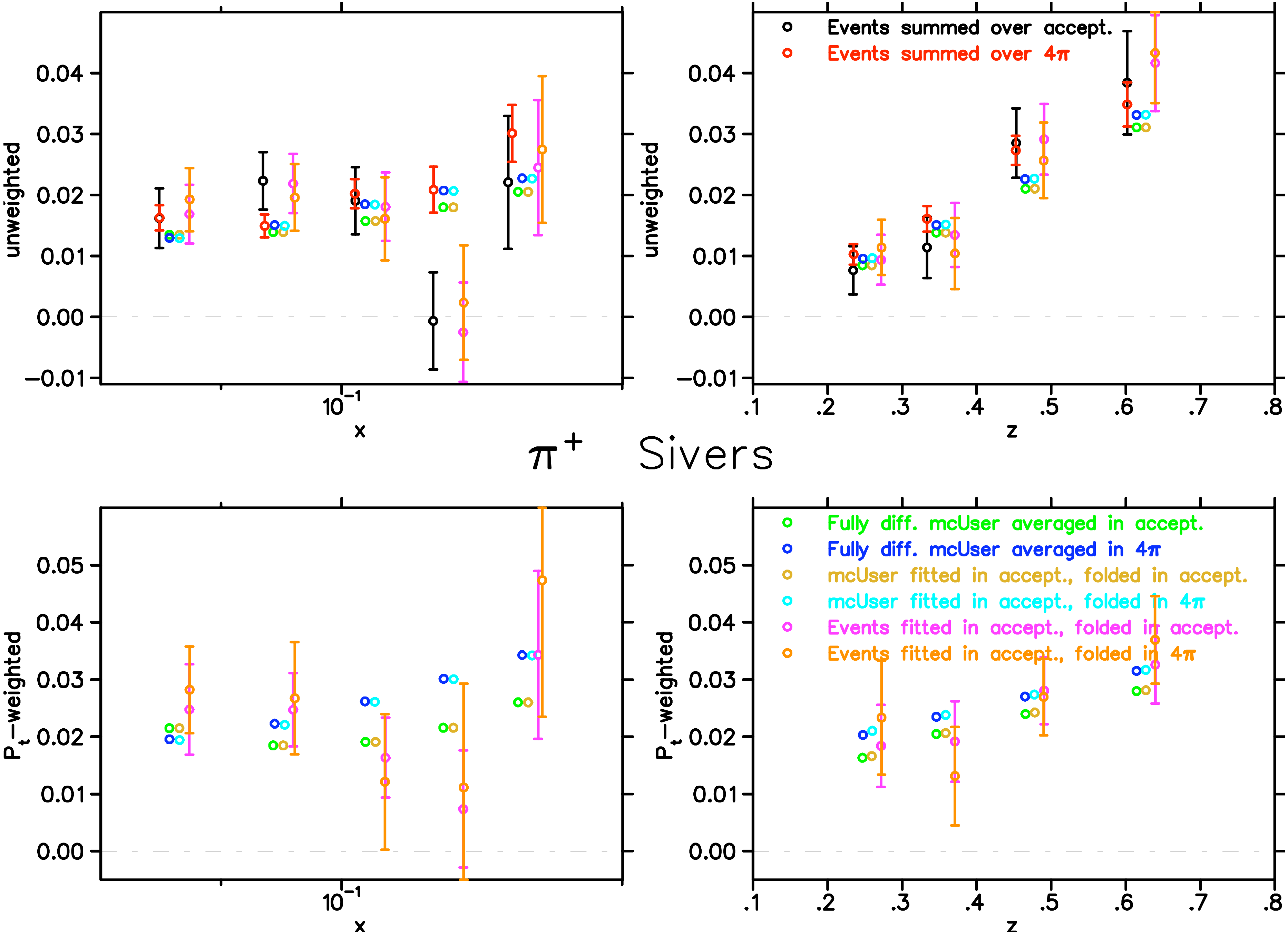

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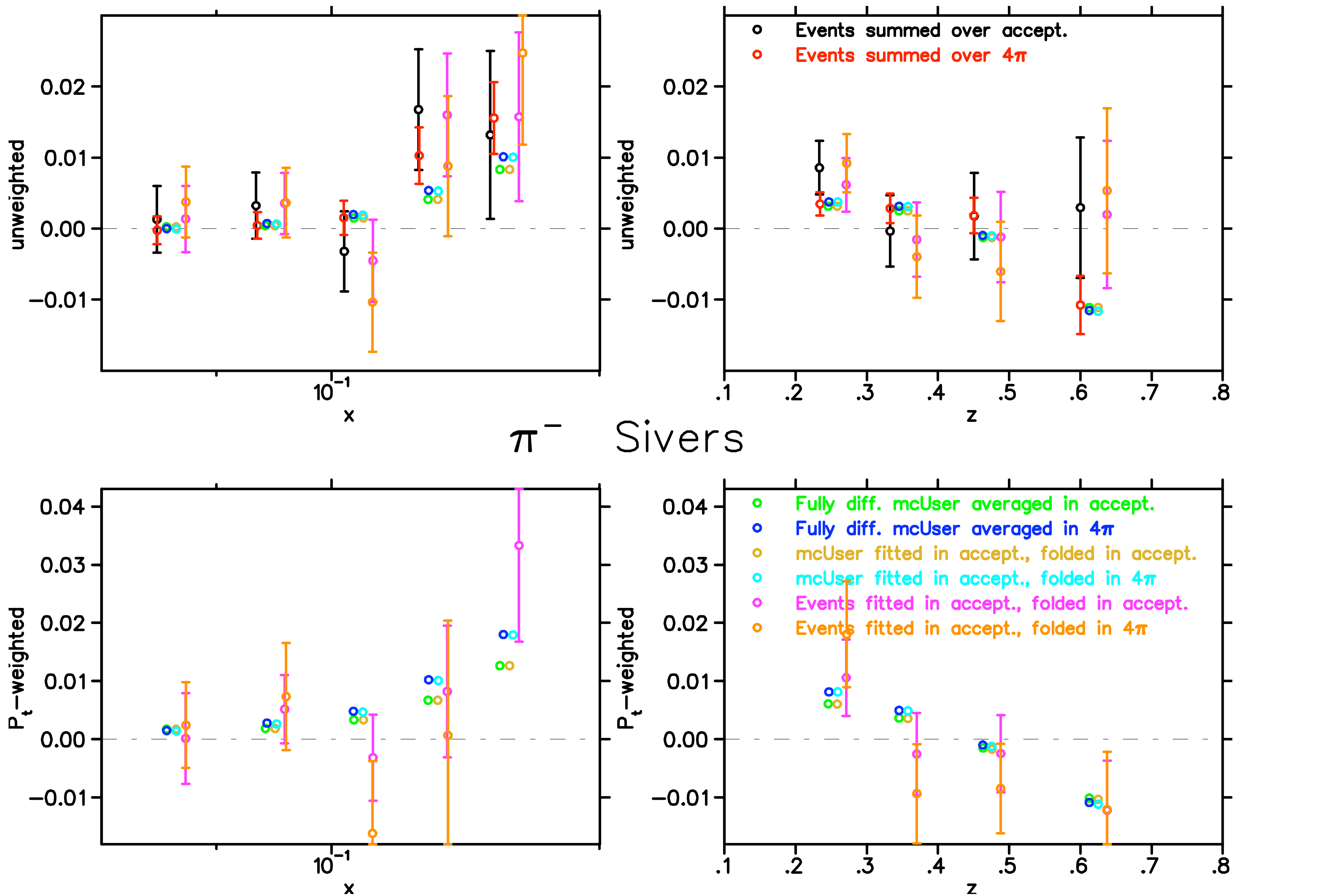
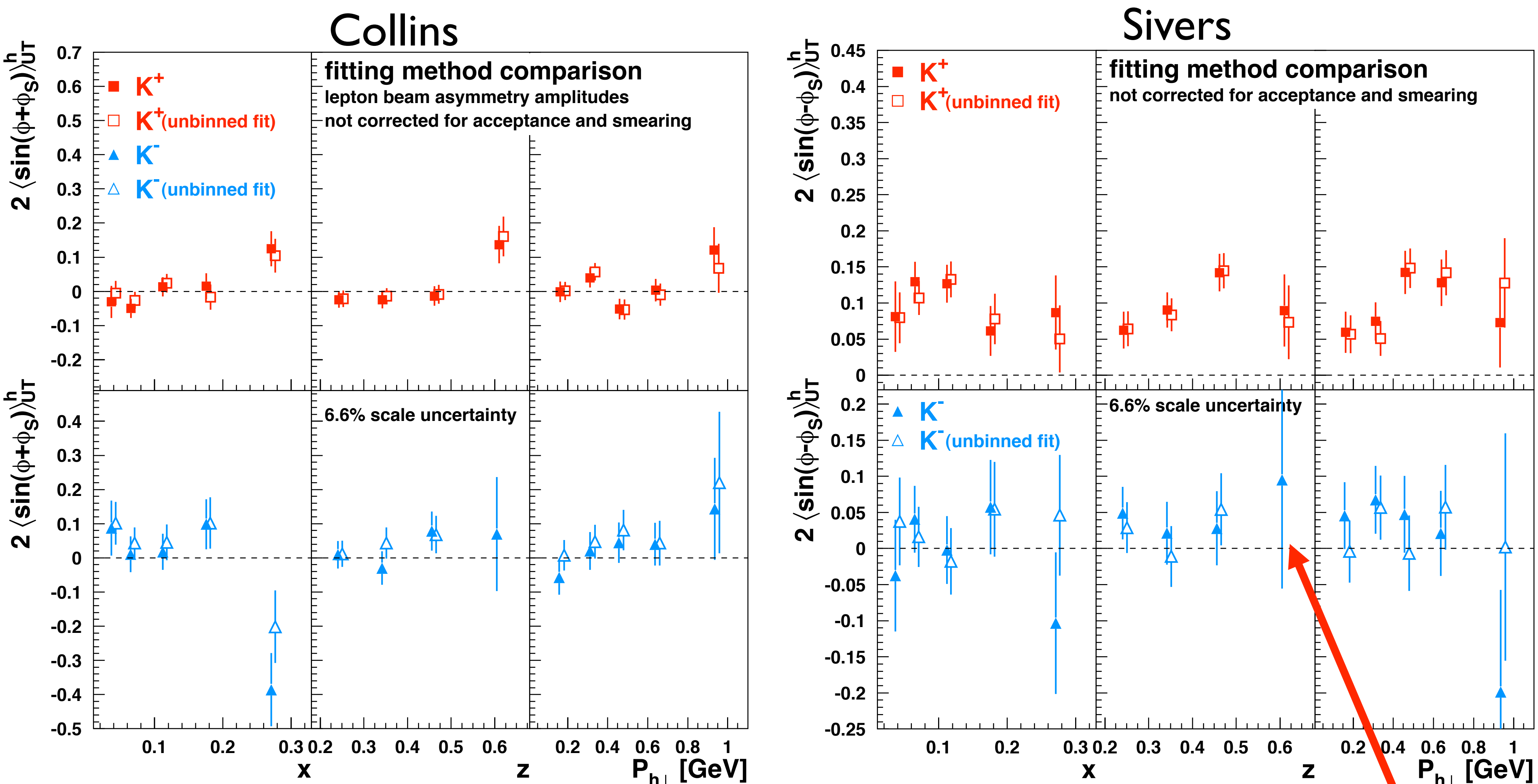


Summary

-
- An unknown systematic error from multi-dimensional correlations between acceptance and asymmetry is exchanged for a slightly larger but well-known statistical uncertainty
- All available information about the correlated kinematic dependence of the asymmetries can be extracted and available for formation of any projection of the result

2-parameter ML fits for Kaons



Filled symbols: Least-χ^2 fit; Empty symbols: Maximum Likelihood

In the last z-bin for Sivers K^-, no likelihood maximum exists.

Did least-χ^2 invent a spurious solution?

Q^2 dependence in 22- or 44-parameter fits of expt'l data

-
- For all charged pions and kaons, the linear coefficient of Q^2 is compatible with zero for both Collins and Sivers fits
- For the Sivers asymmetry of the neutral pions, this coefficient is negative by about 3.5σ ,which means that the generally positive value appears to be power-suppressed.
- Is this in the right direction to "explain" the violation of the Makins Relation?